

WFM03/01: Further Pure Mathematics F3

Question Number	Scheme	Marks
1.	$\pm \frac{a}{e} = 8, \quad \pm ae = 2$ $\frac{a}{e} \times ae = a^2 = 16$ $a = 4$ $b^2 = a^2(1 - e^2) = a^2 - a^2 e^2$ $\Rightarrow b^2 = 16 - 4 = 12$ $\Rightarrow b = \sqrt{12} = 2\sqrt{3}$	B1, B1 B1 M1 A1 (5) 5

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2.	$x^2 + 4x + 13 = (x + 2)^2 + 9$ $\int \frac{1}{(x + 2)^2 + 9} dx = \frac{1}{3} \arctan\left(\frac{x + 2}{3}\right)$ $\left[\frac{1}{3} \arctan\left(\frac{x + 2}{3}\right)\right]_{-2}^1 = \frac{1}{3} (\arctan 1 - \arctan 0)$ $= \frac{\pi}{12}$	B1 M1 A1 M1 A1 (5) 5

Question Number	Scheme	Marks
3(a)	$rhs = 1 + 2 \sinh^2 x = 1 + 2 \left(\frac{e^x - e^{-x}}{2} \right)^2$ $= \frac{2 + e^{2x} - 2 + e^{-2x}}{2}$ $= \frac{e^{2x} + e^{-2x}}{2} = \cosh 2x = lhs \quad *$	M1 M1 A1 (3)
(b)	$1 + 2 \sinh^2 x - 3 \sinh x = 15$ $2 \sinh^2 x - 3 \sinh x - 14 = 0$ $(\sinh x + 2)(2 \sinh x - 7) = 0$ $\sinh x = -2, \frac{7}{2}$ $x = \ln \left(-2 + \sqrt{(-2)^2 + 1} \right) = \ln \left(-2 + \sqrt{5} \right)$ $x = \ln \left(\frac{7}{2} + \sqrt{\left(\frac{7}{2} \right)^2 + 1} \right) = \ln \left(\frac{7 + \sqrt{53}}{2} \right)$	M1 M1 A1 M1 A1 (5) 8

Question Number	Scheme	Marks
4(a)	$\int (a-x)^n \cos x dx = (a-x)^n \sin x + \int n(a-x)^{n-1} \sin x dx$ $\left[(a-x)^n \sin x \right]_0^a = 0$ $= -n(a-x)^{n-1} \cos x - \int n(n-1)(a-x)^{n-2} \cos x dx$ $I_n = na^{n-1} - n(n-1)I_{n-2} \quad *$	M1A1 A1 dM1 A1 (5)
(b)	$I_2 = 2\left(\frac{\pi}{2}\right) - 2\int_0^{\frac{\pi}{2}} \cos x dx$ $= \pi - 2[\sin x]_0^{\frac{\pi}{2}} = \pi - 2$	M1 A1 A1 (3)
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Question Number	Scheme	Marks
5(a)	$\frac{dy}{dx} = 2\operatorname{ar\,cosh}(3x) \times \frac{3}{\sqrt{9x^2-1}}$ $\sqrt{9x^2-1} \frac{dy}{dx} = 6\operatorname{ar\,cosh}(3x)$ $(9x^2-1) \left(\frac{dy}{dx}\right)^2 = 36(\operatorname{ar\,cosh}(3x))^2$ $(9x^2-1) \left(\frac{dy}{dx}\right)^2 = 36y \quad *$	M1A1A1 dM1 A1 (5)
(b)	$\left\{ 18x \left(\frac{dy}{dx}\right)^2 + (9x^2-1) \times 2 \frac{dy}{dx} \times \frac{d^2y}{dx^2} \right\} = 36 \frac{dy}{dx}$ $(9x^2-1) \frac{d^2y}{dx^2} + 9x \frac{dy}{dx} = 18 \quad *$	M1 {A1} A1 A1 (4) 9

Question Number	Scheme	Marks
6(a)	$\begin{pmatrix} 1 & 0 & 3 \\ 0 & -2 & 1 \\ k & 0 & 1 \end{pmatrix} \begin{pmatrix} 6 \\ 1 \\ 6 \end{pmatrix} = \lambda \begin{pmatrix} 6 \\ 1 \\ 6 \end{pmatrix}$ $\begin{pmatrix} 24 \\ 4 \\ 6k+6 \end{pmatrix} = \begin{pmatrix} 6\lambda \\ \lambda \\ 6\lambda \end{pmatrix}$ <p>Uses the first or second row to obtain $\lambda = 4$</p>	M1A1 (2)
(b)	<p>Uses the third row and their $\lambda = 4$ to obtain</p> $6k+6=24 \Rightarrow k=3 \quad *$	M1 A1 (2)
(c)	$\begin{vmatrix} 1-\lambda & 0 & 3 \\ 0 & -2-\lambda & 1 \\ 3 & 0 & 1-\lambda \end{vmatrix} = 0$ $\Rightarrow (1-\lambda)((-2-\lambda)(1-\lambda)-0)-0(0(1-\lambda)-3)+3(0-3(-2-\lambda))=0$ $\Rightarrow (1-\lambda)(-2-\lambda)(1-\lambda)+9(2+\lambda)=(2+\lambda)(9-(1-\lambda)^2)=0 \quad (\lambda^3-12\lambda-16=0)$ $\Rightarrow (\lambda+2)(\lambda^2-2\lambda-8)=0$ $\Rightarrow (\lambda+2)(\lambda+2)(\lambda-4)=0$ $\lambda = -2, 4$	M1 A1 M1 A1 (4)
(d)	<p>Parametric form of $l_1 : (t+2, -3t, 4t-1)$</p> $\begin{pmatrix} 1 & 0 & 3 \\ 0 & -2 & 1 \\ 3 & 0 & 1 \end{pmatrix} \begin{pmatrix} t+2 \\ -3t \\ 4t-1 \end{pmatrix} = \begin{pmatrix} 13t-1 \\ 10t-1 \\ 7t+5 \end{pmatrix}$ <p>Cartesian equations of $l_2 : \frac{x+1}{13} = \frac{y+1}{10} = \frac{z-5}{7}$</p>	M1 M1 A1 ddM1A1(5)
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Question Number	Scheme	Marks
7(a)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 1 & 0 \\ 6 & -2 & 1 \end{vmatrix} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} = 5$ $\mathbf{r} \cdot \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} = 5$	M1 A2(1,0)
(b)	<p>Equation of l is $\mathbf{r} = \begin{pmatrix} 6 \\ 13 \\ 5 \end{pmatrix} + t \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$</p> <p>At intersection $\begin{pmatrix} 6+t \\ 13+4t \\ 5+2t \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} = 5$</p> $\Rightarrow 6+t+4(13+4t)+2(5+2t)=5 \Rightarrow t=-3$ <p>N is $(3,1,-1)$ *</p>	M1 M1 M1 A1 (4)
(c)	$\overline{PN} \cdot \overline{PR} = (-3\mathbf{i} - 12\mathbf{j} - 6\mathbf{k}) \cdot (-5\mathbf{i} - 13\mathbf{j} - 3\mathbf{k}) = 189$ $\sqrt{9+144+36} \sqrt{25+169+9} \cos NPR = 189$ $NX = NP \sin NPR = \sqrt{189} \sin NPR = 3.61$	M1 A1ft A1 M1A1 (5)
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Question Number	Scheme	Marks
8(a)	$\frac{dx}{dt} = 4 \sec t \tan t \quad \frac{dy}{dt} = 2 \sec^2 t$ $\frac{dy}{dx} = \frac{2 \sec^2 t}{4 \sec t \tan t} \quad \left(= \frac{1}{2 \sin t} \right)$ $y - 2 \tan t = \frac{1}{2 \sin t} (x - 4 \sec t)$ $2y \sin t - \frac{4 \sin^2 t}{\cos t} = x - \frac{4}{\cos t}$ $2y \sin t = x - \frac{4 - 4 \sin^2 t}{\cos t} = x - 4 \cos t \quad *$	B1 (both) M1 M1 A1 A1 (5)
(b)	Gradient of l_2 is $-2 \sin t$ $y = -2x \sin t \quad (2)$ $2(-2x \sin t) \sin t = x - 4 \cos t \Rightarrow x = \frac{4 \cos t}{1 + 4 \sin^2 t} \quad (1)$ $y = \frac{-8 \sin t \cos t}{1 + 4 \sin^2 t}$ $(x^2 + y^2)^2 = \left(\frac{16 \cos^2 t}{(1 + 4 \sin^2 t)^2} + \frac{64 \sin^2 t \cos^2 t}{(1 + 4 \sin^2 t)^2} \right)^2$ $= \frac{256 \cos^4 t}{(1 + 4 \sin^2 t)^4} (1 + 4 \sin^2 t)^2 = \frac{256 \cos^4 t}{(1 + 4 \sin^2 t)^2}$ $16x^2 - 4y^2 = \frac{256 \cos^2 t}{(1 + 4 \sin^2 t)^2} - \frac{256 \sin^2 t \cos^2 t}{(1 + 4 \sin^2 t)^2} = \frac{256 \cos^4 t}{(1 + 4 \sin^2 t)^2}$	M1 A1 M1 A1 M1 A1 M1 A1 (8) 13